

NAME:

SID:

This homework asks you to provide some answers that involve math and some that involve code.

• **Math:** Your answers should be written just as they would be in a class where you weren't using a computer. You can leave answers unsimplified unless the question says otherwise. For example, answers like  $(1/2) + (1/2)^3$  or  $(364/365)(363/365) \cdots (350/365)$  are fine.

• **Code:** One of the most effective ways to improve your ability to program is to think through your first draft of a program using pencil and paper. Try to solve these questions without a computer. You can use a computer afterwards to check your work.

### Problem 1

A student has written the following Python code:

```
beatles = ['John', 'Paul']  
beatles + 'George' + 'Ringo'
```

hoping to create the list

```
['John', 'Paul', 'George', 'Ringo']
```

Has the student succeeded? If not, edit or rewrite the statements so that you use `beatles` and create the correct list for the student.

### Problem 2

A Python array called `incomes` consists of 213 incomes arranged in increasing order. You can assume that all the incomes are different from each other, and that the `numpy` module has been imported as `np`. Write Python expressions that generate:

- an array consisting of the integers 1 through 213
- the second-largest income
- the total of all the incomes
- a boolean that answers the question, "Is the largest income at least as large at 10 times the smallest income?"
- the smallest difference between any two incomes
- the maximum number of incomes you can sum together without exceeding 1000000 [Hint: You can cumulatively sum all elements of an array using `np.cumsum` and count how many `True` values appear in a boolean array using `np.count_nonzero`.]

### Problem 3

A *random number generator* draws repeatedly at random from among the 10 digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9, in such a way that each digit is equally likely to be drawn each time, regardless of the results of the other draws.

- (a) Write a Python expression for  $(9/10)^6 \times (1/10)$ .
- (b) The expression in part (a) is equal to the chance that (pick one option and explain your choice):
- (i) the digit 0 appears on the 7th draw
  - (ii) the digit 0 appears exactly once in 7 draws
  - (iii) the digit 0 appears for the first time on the 7th draw
  - (iv) it takes more than 6 draws for the digit 0 to appear

### Problem 4

A monkey is banging repeatedly on the keys of a typewriter. Each time, the monkey is equally likely to hit any of the 26 letters of the English alphabet, regardless of what it hits at other times. There are no other keys on the keyboard. Find a numerical expression (no code) for the chance that:

- (a) the monkey types the sequence `datascience`
- (b) the first ten letters that the monkey types are all different
- (c) there is at least one `e` among the first four letters that the monkey types

### Problem 5

Use the function `np.arange` to write a Python expression for the answer to part (b) in the problem above.

### Problem 6

Let  $N$  be an even positive integer. Qualitatively describe the result of the following expressions in terms of  $N$  (for example, “a tuple consisting of the first  $N$  even numbers”).

- (a) `(np.arange(1, N+1))**2`
- (b) `np.cumsum(np.arange(1, 2*N+1, 2))`