# Review II, December 7

DATA 8
Fall 2016

## Inference; Theory of Prob/Stat

Slides created by Ani Adhikari and John DeNero

# Plan for This Week

- **Today**:
    - Complete Inference review
    - Theory of Prob/Stat
- GSIs: **today and tomorrow during lab times**:
    - First hour: review problems on particular topic
    - Second hour: office hour
    - Topics on Review links on data8.org
- Fri: Go see a dumb movie or relax in some other way

# Final Exam

- **Monday December 12, 8:00 - 11:00**
- **RSF Field House**
- Bring something to write with and something to erase with; but not your breakfast. Water is OK.
- We will provide a couple of reference sheets, with drafts posted on Piazza during RRR week
- 16 questions (six 5-pointers, five 6-pointers, five 8-pointers).
- Covers the whole course

# Big Picture of Course Contents

1. Python
2. Describing data
3. General concepts of inference
4. Theory of probability and statistics
5. Methods of inference

# 5. … Continued from Last Time

Inference: Tests of Hypotheses

# Comparing Two Categorical Samples

- **Null:** The two samples come from the same underlying distribution in the population
- Test statistic: TVD between the distributions of the two samples
- **Method:**
  - Permutation: Under the null, pool the two samples, shuffle, and split into new samples A and B

- 16.1 (mitoses rating CKD/non-CKD; clump thickness rating cancerous/non-cancerous)

# Comparing Two Numerical Samples

- **Null:** The two samples come from the same underlying distribution in the population.
- Test statistic: difference between sample means (take absolute value depending on alternative)
- **Methods (two!) for A/B Testing:**
  - Permutation under the null: 10.4 (Deflategate), 16.2 (birth weight etc for smokers/nonsmokers), 16.3 (BTA RCT)
  - Bootstrap CI for difference: 16.2, 16.3

# Causality

- Tests of hypotheses can help decide that a difference is not due to chance

- But they don't say ***why*** there is a difference …

- Unless the data are from an RCT 16.3
  - In that case a difference that's not due to chance can be ascribed to the treatment

# Classification

- Binary classification based on attributes 15.1
  - *k*-nearest neighbor classifiers
- Training and test sets 15.2
  - Why these are needed
  - How to generate them
- Implementation: 15.4
  - Distance between two points
  - Class of the majority of the *k* nearest neighbors
- Accuracy: Proportion of test set correctly classified 15.5

# 4. Probability and Statistics: Theory

- Descriptive statistics:
  - One variable
  - Two variables
- Probability theory:
  - Exact calculations
  - Normal approximation for mean of large random sample
  - Accuracy and sample size

# Measures of Center

- Median: 50th percentile, where
  - $p$th percentile = smallest value on list that is at least as large as $p$% of the values 11.1
- Median is not affected by outliers
- Mean of 5, 7, 8, 8 = (5+7+8+8)/4 12.1

  = 5*0.25 + 7*0.25 + 8*0.5
- Mean depends on all the values; smoothing operation; center of gravity of histogram; if histogram is skewed, mean is pulled away from median towards the tail

# Measure of Spread

**Standard deviation** (SD)

=

| | | | | |
|---|---|---|---|---|
| root<br>5 | mean<br>4 | square of<br>3 | deviations from<br>2 | average<br>1 |

Measures roughly how far off the values are from average

- 12.2

# Chebychev's Bounds

| Range | Proportion |
|---|---|
| average ± 2 SDs | at least 1 - 1/4 (75%) |
| average ± 3 SDs | at least 1 - 1/9 (88.888…%) |
| average ± 4 SDs | at least 1 - 1/16 (93.75%) |
| average ± $z$ SDs | at least 1 - $1/z^2$ |

**no matter what the distribution looks like**

# How Big are Most of the Values?

***No matter what the shape of the distribution,***
the bulk of the data are in the range “average ± a few SDs”

***If a histogram is bell-shaped***, then

- the SD is the distance between the average and the points of inflection on either side
- Almost all of the data are in the range “average ± 3 SDs”

12.2, 12.3

# Bounds and normal approximations

| Percent in Range | All Distributions | Normal Distribution |
|---|---|---|
| average $\pm$ 1 SD | at least 0% | about 68% |
| average $\pm$ 2 SDs | at least 75% | about 95% |
| average $\pm$ 3 SDs | at least 88.888...% | about 99.73% |

# Standard Units z

“average ± $z$ SDs” 12.2

- $z$ measures “how many SDs above average”
- Almost all standard units are in the range (-5, 5)
- To convert a value to standard units:

$$z = \frac{\text{value - average}}{\text{SD}}$$

# Definition of *r*

**Correlation Coefficient** (*r*) =

| | | | | |
|---|---|---|---|---|
| average of | product of | *x* in standard units | and | *y* in standard units |

Measures how clustered the scatter is around a straight line

# The Correlation Coefficient *r*

- Measures ***linear*** association
- Based on standard units; pure number with no units
- *r* is not affected by changing units of measurement
- $-1 \leq r \leq 1$
- $r = 0$: No linear association; *uncorrelated*
- *r* is not affected by switching the horizontal and vertical axes
- 13.1

# Regression to the Mean

- **estimate of $\boldsymbol{y = r \cdot x}$**, when both variables are measured in standard units
- If $r = 0.6$, and the given $x$ is 2 standard units, then:
  - The given $x$ is 2 SDs above average
  - The prediction for $y$ is 1.2 SDs above average
- On average (though not for each individual), regression predicts $y$ to be closer to the mean than $x$ is
- 13.2

# Regression Estimate, Method I

A course has a midterm (average 70; standard deviation 10) and a really hard final (average 50; standard deviation 12)

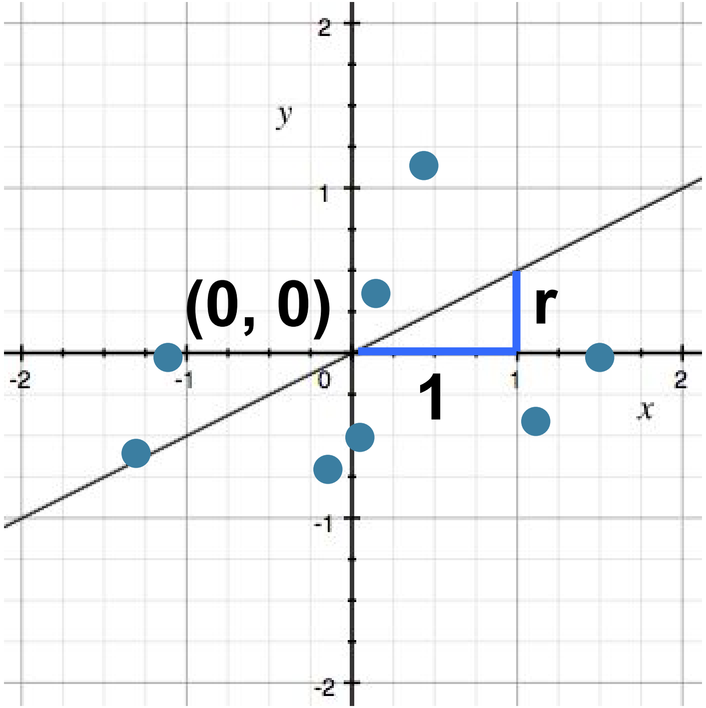
If the scatter of midterm & final scores for students looks like a typical oval with correlation 0.75, then...

What do you expect the average final score would be for a student who scored 90 on the midterm?

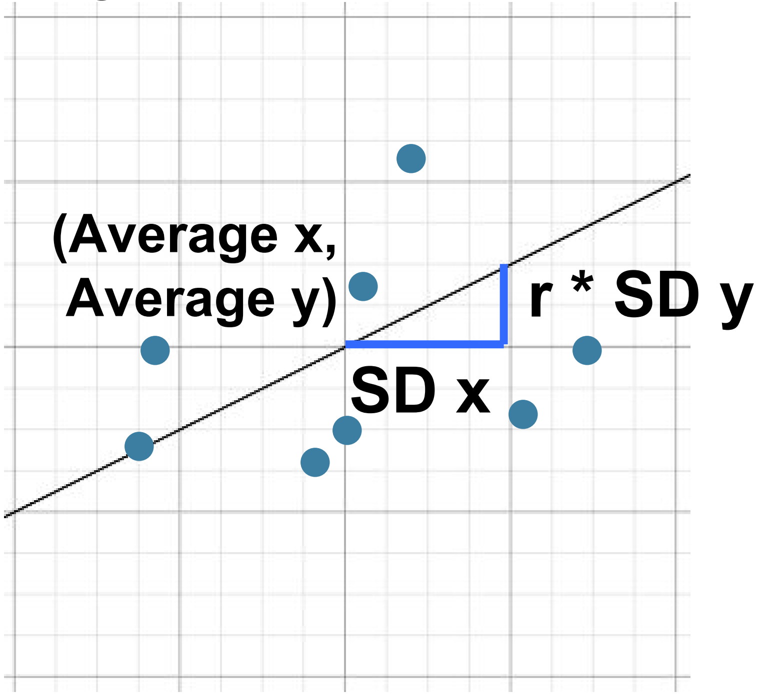
2 standard units on midterm,
so estimate 0.75 * 2 = 1.5 standard units on final.
So estimated final score = 1.5 * 12 + 50 = 68 points

# Regression Line

Standard Units




Original Units



# Slope and Intercept

estimate of $y$ = slope * $x$ + intercept

$$\textbf{slope of the regression line} = r \cdot \frac{\text{SD of } y}{\text{SD of } x}$$

$$\textbf{intercept of the regression line} = \text{average of } y - \text{slope} \cdot \text{average of } x$$

- 13.2

# Regression Estimate, Method II

The equation of a regression line for estimating child's height based on midparent height is

estimated child's height = 0.64·midparent height + 22.64

Estimate the height of someone whose midparent height is 69 inches.

0.64*69 + 22.64 = 66.8 inches

# Residuals

- Error in regression estimate
- One residual corresponding to each point ($x$, $y$)
- **residual = observed $y$ - regression estimate of $y$**

  = vertical difference between point and line

- No matter what the shape of the scatter plot:
  - Residual plot does not show a trend
  - Average of residuals = 0

$$\text{SD of residuals} = \sqrt{1 - r^2} \times \text{SD of } y$$

# Equally Likely Outcomes

- **If all outcomes are assumed equally likely**, then probabilities are proportions of outcomes:

$$P(A) = \frac{\text{number of outcomes that make A happen}}{\text{total number of outcomes}}$$

$$= \text{proportion of outcomes that make A happen}$$

- 8.4

# Probability: Exact Calculations

- Probabilities are between 0 (impossible) and 1 (certain)
- P(event happens) = 1 - P(the event doesn't happen)
- Chance that two events *A* and *B* both happen

= P(*A* happens) x P(*B* happens given that *A* has happened)

- If event *A* can happen in *exactly one* of two ways, then

  P(*A*) = P(first way) + P(second way)
- 8.4

# Updating Probabilities

- Start with **prior probabilities** of two classes; priors can be **subjective**
- Known: **likelihood** of data, given each of the classes
- Acquire data according to these likelihoods
- Update the prior probabilities by finding **posterior probabilities** of the two classes, **given the data**
- Tree diagrams and **Bayes' Rule:** 17.1, 17.2

# Approximation: CLT

## Central Limit Theorem

If the sample is

- large, and
- drawn at random with replacement,

Then, *regardless of the distribution of the population,*

**the probability distribution of the sample sum**
**(or of the sample mean) is *roughly* bell-shaped** 12.4

# Random Sample Mean

- Fix a sample size
- Draw ***all possible random samples*** of that size
- Compute the mean of each sample
- You'll end up with a lot of means
- The distribution of those is the *probability distribution of the sample mean*
- It's centered at the population mean
- SD = (population SD)/$\sqrt{\text{(sample size)}}$ 12.5
- If the sample is large, it's roughly bell shaped by CLT

# Accuracy of Random Sample Mean

- Greater if SD of sample mean is smaller
- Doesn't depend on population size
- Increases as sample size increases, because SD of sample mean decreases
- For 3 times the accuracy, you have to multiply the sample size by a factor of $3^2 = 9$
- **Square Root Law:** If you multiply sample size by a factor, accuracy goes up by the square root of the factor
- 12.5

# Application to Proportions

- Fact: **SD of 0-1 population $\leq$ 0.5** 12.6
- Total width of 95% CI for population proportion:

  $= 4$ SDs of the sample proportion

  $= 4 \times (\text{SD of 0-1 population})/\sqrt{(\text{sample size})}$

  $\leq 4 \times 0.5/\sqrt{(\text{sample size})}$

  $= 2 / \sqrt{(\text{sample size})}$
- So if you know the desired width of the interval, you can solve for (an overestimate of) the sample size